Exam #1 cheat sheet

- Other than this cheat sheet (which you should tear off), all you are allowed to use for help are the basic functions on a calculator.
- The space provided below each question should be sufficient for your answer, but you can use additional paper if needed.
- *Show your work for partial credit.* It is very difficult to give partial credit if the only thing on your page is "$x = 3$".
- **Expected value** is given by summing likelihood times value over all possible outcomes:

$$\text{Expected Value} \quad = \sum_{\text{Outcomes } i} \text{Probability}(i) \cdot \text{Value}(i).$$

- A **fair bet** is a bet with an expected value of zero.
- The **future value of a lump sum payment** of $\$x$ invested for n years at interest rate r is $\text{FV} = x(1+r)^n$. The **present value of a lump sum payment** of $\$x$ after n years at interest rate r is $\text{PV} = \dfrac{x}{(1+r)^n}$. (Note that this formula also works for values of n that are negative or zero.)
- The present value of an **annuity** paying $\$x$ at the end of each year for n year at interest rate r is

$$\text{PV} = x \left[\frac{1 - \dfrac{1}{(1+r)^n}}{r} \right].$$

  The present value of the related **perpetuity** (with annual payments forever) is

$$\text{PV} = \frac{x}{r}.$$

- The **inflation rate**, i, is the rate at which prices rise. The **nominal interest rate**, r_N, is the interest rate in terms of dollars. The **real interest rate**, r_R, is the interest rate in terms of purchasing power. These are related by

$$1 + r_R = \frac{1 + r_N}{1 + i}.$$

  When the inflation rate is small, we can approximate this as

$$r_R \approx r_N - i.$$

Name:

1. (5 points) Use **sunk cost** in a sentence that relates it to a real-life example about climate change or another environmental issue. (For partial credit you can use a non-environmental example, or define sunk cost, or otherwise discuss the issue.) It may help to draw a decision tree, but you are not required to.

2. Any type of climate change policy has to think about enforcement; for a cap-and-trade policy, the enforcement approach is to fine firms that emit CO_2 without having the appropriate number of permits. Imagine that a profit-maximizing firm has a choice between buying the appropriate permits (for, say, \$20 per ton of CO_2) or risking it and getting a fine. If the firm takes the risk, imagine that there's an 70% chance that it will get away with it (and hence pay nothing) and a 30% chance that it will have to pay a fine of \$90 per ton of CO_2.

   (a) (5 points) The expected amount of money the firm will pay if they buy the appropriate permits is, obviously, \$20 per ton of CO_2. What is the expected amount of money they'll pay if they risk it?

   (b) (5 points) One way for the government to make sure that firms will buy the required permits to increase inspections so that firms that cheat get caught more often. Use your expected value calculation above to suggest another way to accomplish this same goal.

3. The EPA (Environmental Protection Agency) and the CBO (Congressional Budget Office) have both done analyses of the Waxman-Markey cap-and-trade bill (HR 2454) that estimate permit prices for the next few decades. The EPA estimates that permit prices in 2020 will be about \$18 per metric tonne of CO_2 *in 2005 dollars*. The CBO estimates that permit prices in 2020 will be about \$28 per metric tonne of CO_2 *in 2020 dollars*. Assume that inflation in the 15 years between 2005 and 2020 is expected to be 2% per year.

   (a) (5 points) Translate both the EPA and the CBO estimates into 2009 dollars so we can see which one is larger. *Show your work.*

   (b) (5 points) Explain (as if to a non-economist) what it means that the CBO estimate for permit prices in 2020 is \$28 *in 2020 dollars*.

   (c) (5 points) Explain (as if to a non-economist) what it means that the CBO estimate for permit prices in 2020 is \$X *in 2009 dollars*.

   (d) (5 points) One gallon of gasoline produces about 20 pounds of CO_2, and there are about 2,205 pounds in a metric tonne. How much does a tax of (say) \$20 per metric tonne of CO_2 translate into in terms of dollars per gallon of gasoline?

4. Geothermal energy involves "mining" heat by drilling into the earth's crust. Like many clean energy technologies, it has high up-front costs but promises to pay off over time. The made-up numbers in this problem look at the economics of geothermal power. (For the real numbers, see the 2007 MIT report "The Future of Geothermal Energy".)

   (a) (5 points) Consider spending $1000 today to build a geothermal plant that will generate $100 at the end of each year for the next 30 years. Show that the present value of the costs outweigh the present value of the benefits if the interest rate is 13%.

   (b) (5 points) In order to make geothermal more attractive, does the interest rate need to go up or down? Briefly explain.

   (c) (5 points) What will the present value of benefits be if the plant generates $100 a year *forever* instead of just for 30 years? (The interest rate is still 13%.)

   (d) (5 points) Explain (as if to a non-economist) why the present value of the $100-a-year stream of benefits will not be infinite even if the plant can operate forever. Do *not* talk about inflation.

5. Imagine that you own a lake and that you're trying to maximize the present value of catching fish from the lake, which currently has 1000 fish in it. The population growth function of the fish is described in Figure 1.

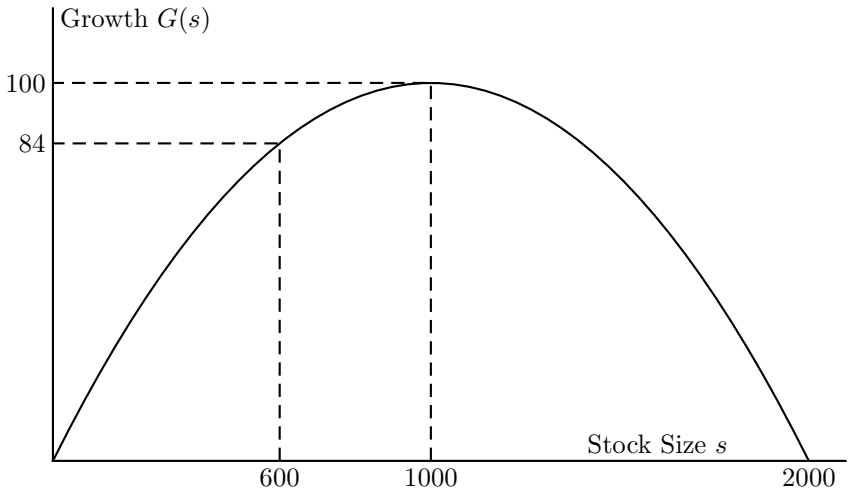

Figure 1: A population growth function for fish.

(a) (5 points) The maximum sustainable yield policy is to catch 100 fish at the end of this year, 100 fish at the end of the next year, and so on, forever. If the price of fish is always \$1 per fish, what is the resulting present value at a 5% interest rate?

(b) (5 points) An alternative policy is to catch 400 fish *today* (so that 600 remain in the lake), and then catch 84 fish at the end of this year, 84 fish at the end of the next year, and so on, forever. What is the resulting present value? (Assume as above a price of \$1 per fish and an interest rate of 5%.) Is it higher or lower than the present value of the maximum sustainable yield policy?

6. Imagine that you are a profit-maximizing forester. You currently own trees containing 100 board-feet of timber.

   (a) (5 points) With probability 2%, a fire will destroy your trees, and you'll have no harvestable timber. With probability 98%, your trees will grow and in one year you'll have 5% more board-feet of timber. What is the expected number of board-feet of timber you'll have next year?

   (b) (5 points) Explain (as if to a non-economist) why the interest rate at the bank matters in deciding to cut the trees down now or to cut them down in year. (Hint: "trees are capital.")

7. "Opportunities for arbitrage are self-eliminating."

   (a) (5 points) Explain this statement in the context of lanes of traffic on a congested freeway or attempts by "active investors" to pick stock-market winners.